LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **STATISTICS**

THIRD SEMESTER – NOVEMBER 2007

ST 3875 - FUZZY THEORY AND APPLICATIONS

BB 4

Date : 01/11/2007 Dept. No. Max. : 100 Marks

Time : 9:00 - 12:00

SECTION - A

Answer ALL the Questions (10 x 2 = 20 marks)

1. Define a fuzzy t-norm.
2. What is bounded sum and bounded difference? Explain
3. When do you say a t-norm and a t-conorm are dual with respect to each other.
4. What are the two methods for defining fuzzy arithmetic?
5. How do you define arithmetic operations on intervals?
6. Give an example of a membership function for the 'Set of real numbers close to zero'.
7. Define 'Support' and 'Core' of a fuzzy set.
8. State the 'Axiomatic Skeleton' for fuzzy complements.
9. Briefly explain the 'direct method with one expert' for constructing membership functions.
10. What is an 'activation function'? State some basic types of activation functions.

SECTION - B

Answer any FIVE Questions (5 x 8 = 40 marks)

11. (a) Prove that the standard fuzzy intersection is the only idempotent t-norm.
    (b) Prove that $i_{\min}(a,b) \leq i(a,b) \leq \min(a,b)$, for all a,b $\in$ [0, 1]. (4 + 4)
12. Given $g(a) = \begin{cases} \dfrac{a+1}{2} & ; \quad a \neq 0 \\ 0 & ; \quad a = 0 \end{cases}$

    Determine $i^g(a,b) = g^{-1}(i(g(a), g(b)))$.
13. State and prove the law of excluded middle and law of contradiction.
14. Let A and B be two fuzzy numbers. If

    $^{\alpha}(A.B) = [-4\alpha^2 + 12\alpha - 5, 4\alpha^2 - 16\alpha + 15]$ for $\alpha \in [0, 0.5]$

    $[4\alpha^2 - 1, 4\alpha^2 - 16\alpha + 15]$ for $\alpha \in$ [.5, 1].

    Determine the product fuzzy number (A . B).
15. Define the standard complement, union and intersection of fuzzy sets. Give a rough graphical depiction of these operations.
16. Show that every fuzzy complement has at most one equilibrium. Hence, show that a continuous fuzzy complement has a unique equilibrium.
17. Explain the Lagrange's interpolation method of constructing membership function from a sample data. Point out the merits and demerits of the approach.

18. Describe the model of a neuron with its elements and present the functional equations governing the inputs and the output.

SECTION -C

Answer any TWO Questions (2 x 20 = 40 marks)

19. (a) Let i be a t norm and g be a function from $[0, 1] \rightarrow [0, 1]$ strictly increasing and continuous in (0, 1) such that g(0) = 0 and g(1) = 1. Prove that

$$i^g(a, b) = g^{-1}(i\,(\,g(a), g(b)))\ \forall\, a, b \in [0,1] \text{ is a t norm.}$$

(b) Prove the sub-distributive property for fuzzy numbers. (16 + 4)

20. a) Let A and B be fuzzy numbers. Prove that $A * B(z) = \sup\limits_{z=x*y} \min(A(x), B(y))$ is also a fuzzy number, where * is one of the basic arithmetic operations.

(b) Suppose MIN and MAX are binary operations on fuzzy numbers then show that associative and absorption properties hold good. (10+10)

21. (a) State and prove a necessary and sufficient condition for convexity of fuzzy sets.

(b) Let X ={x_1, ..,x_5} be a universal set and suppose three experts E1, E2, E3 have specified the valuations of these five as elements of two fuzzy sets A and B as given in the following table:

Membership in A

Element	E1	E2	E3
x_1	0.2	0.8	0.9
x_2	0.9	0.1	0.2
x_3	0.8	0.3	0.7
x_4	0.1	0.4	0
x_5	0.7	0.5	0.6

Membership in B

Element	E1	E2	E3
x_1	0.8	0.7	0.5
x_2	0.2	0.1	0.4
x_3	0.5	0.6	0.7
x_4	0.4	0.3	0.6
x_5	0.3	0.5	0

Assuming that for set A, the evaluations by the three experts have to be given weights as $c_1 = 1/3$, $c_2 = 1/2$, $c_3 = 1/6$ and for set B as equal weights, find the degree of membership of the five elements in A and in B. Also, evaluate the degree of membership in A∩B using the **bounded difference** operator and in AUB using the **Algebraic sum** operator. (10 +10)

22. (a)Briefly explain the three fundamental problems of ‘Pattern Recognition’.

(b)Present the problem of ‘Fuzzy Clustering’ and explain the Fuzzy c-means algorithm. (6 + 14)

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